Nonlinear Dynamics in the EEG Analysis: Disappointments and Perspectives

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Abstract

Several recent thorough studies have confirmed a nonlinear component in the EEG dynamics, however, signatures of low-dimensional chaos were not found. These results pose the question about adequacy of applying so called chaotic measures (dimensions, Lyapunov exponents) in EEG analysis. It is shown that the chaotic measures applied to stochastic or even noisy chaotic data do not bring information not accessible by linear approaches such as spectral analysis. Moreover, even states of chaotic systems can be discernible by using an entropy rate computed from spectral densities. The applications of the chaotic measures do not seem to lead to a previously expected progress in the computerized EEG analysis, however, there are still ideas and tools developed in study of nonlinear (chaotic) systems, which could contribute to understanding the EEG dynamics and underlying brain processes as well as to improvement of clinical diagnostics. Perspectives for nonlinear dynamics in the computerized EEG analysis are seen in detection and characterization of nonlinearity in EEG dynamics and search for its physiological significance by comparing analyses of real EEG data and of signals generated by realistic models; in classifying complexity of the EEG signals by using entropy rates; or in detecting and characterizing synchronization of EEG signals recorded from different loci.

1 Introduction

During the last two decades there has been a sustained interest in describing neural processes and brain-signals, especially the electroencephalogram (EEG), within the context of nonlinear dynamics and theory of deterministic chaos (see, e.g., [Rapp *et al.,* 1989], [Başar, 1990], [Jansen, 1991], [Freeman, 1992], for comprehensive reviews). If the nature of analyzed signals was actually low-dimensional, the published results could be of immense importance for theoretical neuroscience and neurological and psychiatric clinical practice. However, confidence of results obtained from experimental data, such as finite dimensions or positive Lyapunov exponents, and reliability of chaos-based algorithms in general, have recently come under question, and alternative methods for identifying possible nonlinear determinism in experimental time series have been proposed (see [Weigend & Gershenfeld, 1993], [Paluš, 1995], and references therein). Employing some of these methods, Paluš [1996a] has detected a nonlinear component in EEG recordings of normal healthy volunteers, however, signatures of low-dimensional chaos were not found. Similar results have been independently reported by Pritchard *et al.* [1995], Rombouts *et al.* [1995] and Stam *et al.* [1995]. Theiler & Rapp [1996], Prichard & Theiler [1994], Theiler *et al.,* [1992] and Casdagli [1992] also rejected low-dimensional chaos and confirmed nonlinearity in the EEG, however, they report only a weak evidence for nonlinearity in normal EEG.

Estimation from time series of descriptive measures such as dimensions, Lyapunov exponents or Kolmogorov entropy, derived from theory of deterministic chaos ("chaotic measures") is well established in the case of data generated by low-dimensional deterministic dynamical systems in numerical and laboratory experiments. Questions are naturally raised about applicability of the chaotic measures when analyzing data from real-world systems, which are either stochastic or affected by numerous external influences, which cannot be described in any other way than a stochastic component in system dynamics. Analyzing physiological time series such as the EEG, many authors have realized that low-dimensional chaos in such systems is improbable, however, they have demonstrated that formal estimates of the chaotic measures may possess some discriminating power with respect to data recorded in different experimental conditions [Layne *et al.,* 1986], [Mayer-Kress & Layne, 1987], [Koukkou *et al.,* 1993], [Wackermann *et al.,* 1993]. This "relative characterization" of different datasets may surely have its importance in diagnostics, however, it is questionable whether applications of the chaotic measures for such kind of data is really useful and/or necessary. This question is important from both practical and theoretical points of view. When the chaotic measures, designed for characterization of low-dimensional dynamics, are applied to analysis of high-dimensional or stochastic systems, precision of their estimates, their robustness with respect to noise, or their sensitivity to changes in underlying dynamics can hardly be established. In theoretical aspect, correct interpretation of obtained results is unclear, while using the original meaning and interpretations of the chaotic measures, i.e., using a "low-dimensional language" for high-dimensional or stochastic systems can be misleading.

Searching arguments favorable for applications of the chaotic measures in the EEG analysis, many authors claim that linear (spectral) approaches to the EEG analysis are inadequate because of nonlinear and possibly chaotic character of the EEG. Some researchers consider successful applications of the chaotic measures in a "relative quantification", i.e. an ability to distinguish with a statistical significance EEG signals recorded in different physiological/pathological conditions as an evidence for a chaotic nature of the EEG. In order to demonstrate that the above statements are not correct, we present results of two different, although interconnected studies. In the first study (Sec. 2, for details see [Paluš, 1997a]) it has been found, that the "level of chaos" of chaotic systems is "translated" into their linear properties. More specifically, different states of chaotic systems (states with different Kolmogorov entropy/positive Lyapunov exponents) can be distinguished by an appropriate measure based on power spectra estimated from time series, generated by the chaotic systems. I.e., a linear approach (spectral analysis) can be useful in classification of (nonlinear) chaotic systems. On the other hand, in the second study (Sec. 3, for details see [Paluš, 1998]) we examine the behaviour of the chaotic measures, in particular, of the positive Lyapunov exponents when estimated from a) noisy chaotic data, b) from nonchaotic (linear stochastic) data. It has been found that the estimated Lyapunov exponents failed to distinguish different noisy chaotic time series when relatively small scales were used. The distinction could be reestablished by using larger scales. Using larger scales, however, the estimated Lyapunov exponents are determined by macroscopic statistical properties of the series such as autocorrelations. The latter finding is one of the explanations why formally estimated Lyapunov exponents could lead to (seemingly) successful results, even when applied to nonchaotic data. In such cases the formally estimated Lyapunov exponents would yield random values only for data with equivalent statistical properties, while the so-called successful results (i.e., statistically significant distinction of EEG signals recorded in different physiological/pathological conditions) could be caused by a trivial difference in statistical properties of the data sets, such as different autocorrelations, different ranges of data values, different variances, or different noise levels.

In conclusion, we state that although the chaotic measures do not seem to lead to the previously expected progress in the computerized EEG analysis, there are still ideas and tools developed in study of nonlinear (chaotic) systems, which could contribute to understanding the EEG dynamics and underlying brain processes as well as to improvement of clinical diagnostics. The basic principle to keep is to detect and characterize *real* phenomena present in EEG signals. Thus we see a perspective for nonlinear dynamics in the computerized EEG analysis, e.g., in detection and characterization of nonlinearity (and search for its physiological significance by comparing analyses of real EEG data and of signals generated by realistic models), in classifying complexity of the EEG signals by using entropy rates, or in detecting and characterizing synchronization of EEG signals recorded from different loci.

2 Entropy Rates of Dynamical Systems and Gaussian Processes

Let $\{x_i\}$ be a time series, i.e., a series of measurements done on a system in consecutive instants of time $i = 1, 2, \ldots$. The time series $\{x_i\}$ can be considered as a realization of a stochastic process $\{X_i\}$, characterized by the joint probability distribution function $p(x_1, \ldots, x_n)$, $p(x_1, \ldots, x_n) = \Pr\{(X_1, \ldots, X_n) = (x_1, \ldots, x_n)\}$. The entropy rate of $\{X_i\}$ is defined [Cover & Thomas, 1991] as:

$$h = \lim_{n \to \infty} \frac{1}{n} H(X_1, \ldots, X_n), \tag{1}$$

where $H(X_1, \dots, X_n)$ is the entropy of the joint distribution $p(x_1, \dots, x_n)$:

$$H(X_1, \dots, X_n) = -\sum_{x_1} \dots \sum_{x_n} p(x_1, \dots, x_n) \log p(x_1, \dots, x_n). \tag{2}$$

No general approach to estimating the entropy rates of stochastic processes has been established, except of simple cases such as finite-state Markov chains [Cover & Thomas, 1991]. However, if $\{X_i\}$ is a zero-mean stationary Gaussian process with spectral density function $f(\omega)$, its entropy rate h_G, apart from a constant term, can be expressed using $f(\omega)$ [Chiang, 1987], [Ihara, 1993], [Anh & Lunney, 1995] as:

$$h_G = \frac{1}{2\pi} \int_{-\pi}^{\pi} \log f(\omega) d\omega. \tag{3}$$

Alternatively, the time series $\{x_i\}$ can be considered as a projection of a trajectory of a dynamical system, evolving in some measurable state space. The dynamical complexity of a (chaotic) dynamical system can also be characterized by its entropy rate. As a definition of the entropy rate of a dynamical system, known as the Kolmogorov-Sinai entropy (KSE) [Cornfeld *et al.,* 1982], [Petersen, 1983], [Sinai, 1976] we can consider the equation (1), however, the variables X_i should be understood as m-dimensional variables, according to a dimensionality of the dynamical system [Schuster, 1988]. If the dynamical system is evolving in a continuous measure space, then any entropy depends on a partition chosen to discretize the space and the KSE is defined as a supremum over all finite partitions [Cornfeld *et al.,* 1982], [Petersen, 1983], [Sinai, 1976]. The KSE is a topological invariant, suitable for classification of dynamical systems or their states, and is related to the sum of the system's positive Lyapunov exponents (LE) according to the theorem of Pesin [1977].

Dynamics of a stationary Gaussian process is fully described by its spectrum. Therefore the connection (3) between the entropy rate of such a process and its spectral density $f(\omega)$ is understandable. The estimation of the entropy rate of a Gaussian process is reduced to the estimation of its spectrum.

If a studied time series was generated by a nonlinear, possibly chaotic, dynamical system, its description in terms of a spectral density is not sufficient. Indeed, realizations of isospectral Gaussian processes are used in the surrogate-data based tests in order to discern nonlinear (possibly chaotic) processes from colored noises [Theiler *et al.,* 1992], [Paluš, 1995]. On the other hand, there are results indicating that some characteristic properties of nonlinear dynamical systems may be "projected" into their "linear properties", i.e., into spectra, or equivalently, into autocorrelation functions: Sigeti [1995] has demonstrated that there may be a relation between the sum of positive Lyapunov exponents (KSE) of a chaotic dynamical system and the decay coefficient characterizing the exponential decay at high frequencies of spectra estimated from time series generated by the dynamical system. Asymptotic decay of autocorrelation functions of such time series is ruled by the second eigenvalue of the Perron-Frobenius operator of the dynamical system [Grossmann & Thomae, 1977], [Mori *et al.,* 1981]. Lipton & Dabke [1996] have also investigated asymptotic decay of spectra in relation to properties of underlying dynamical systems.

In a numerical study using a number of discrete and continuous chaotic systems, Paluš [1997a] has investigated a possible relation between the Kolmogorov-Sinai entropy of a dynamical system and the entropy rate (3) of a Gaussian process isospectral to time series generated by the dynamical system, thereafter referred to as "GPER." The results suggest that such a relation as a nonlinear one-to-one function may exist when the Kolmogorov-Sinai entropy varies smoothly with variations of system's parameters, but is broken in critical states near bifurcation points.

For an example of this relation we will consider the discrete baker map, defined as

$$(x_{n+1}, y_{n+1}) = \left(\beta x_n, \frac{1}{\alpha} y_n\right)$$

for $y_n \leq \alpha$, or:

$$(x_{n+1}, y_{n+1}) = \left(0.5 + \beta x_n, \frac{1}{1-\alpha}(y_n - \alpha)\right) \tag{4}$$

for $y_n > \alpha$; $0 \leq x_n, y_n \leq 1$, $0 < \alpha, \beta < 1$, β was set to $\beta = 0.25$. For this system the positive Lyapunov exponent λ_1, or, equivalently, the Kolmogorov–Sinai entropy can be expressed analytically as the function of the parameter α [Farmer *et al.,* 1983], [Hentschel & Procaccia, 1983]:

$$\lambda_1(\alpha) = \alpha \log \frac{1}{\alpha} + (1-\alpha) \log \frac{1}{1-\alpha}. \tag{5}$$

Varying the parameter α from 0.01 to 0.49 with step 0.005, ninety-seven system states with different positive Lyapunov exponents λ_1 were studied. The component y was recorded.[1] In each system state studied, fifteen time series of length 16,384 samples were linearly transformed in order to have zero mean and unit variance[2] and their periodograms[3] were computed using the fast Fourier transform (FFT) [Press *et al.,* 1986]. To prevent numerical underflow, the periodograms were shifted[4] by +1, i.e., $f(\omega) + 1$ was used instead of $f(\omega)$ in Eq. (3).

In Figures 1a-c the comparison of the GPER with the LE (KSE) for the baker map (4) are presented: The LE as the analytic function (5) of the parameter α (Fig. 1a), the GPER, estimated from time series (using their periodograms), plotted against α (Fig. 1b), and the GPER plotted against the LE (Fig. 1c). The latter plot demonstrates that in the case of the chaotic baker map (4) the LE/KSE and the GPER are related by *a nonlinear one-to-one function.* Paluš [1997a] presents also examples of systems in the critical states, when the GPER–KSE relation is broken, here we confine ourselves only to the above demonstration that such a relation exists.

3 The Largest Lyapunov Exponent and Colored Noises

Given a scalar time series $x(t)$, an m-dimensional trajectory is reconstructed using the time-delay method [Takens, 1981] as $\mathbf{x}(t) = \{x(t), x(t+\tau), \ldots, x(t+[m-1]\tau)\}$, where τ is the *delay time* and m is the *embedding dimension.* A neighbour point $\mathbf{x}(t')$ is located so that the initial distance δ_I, $\delta_I = ||\mathbf{x}(t) - \mathbf{x}(t')||$, is $s_{\min} \leq \delta_I \leq \ s_{\max}$. $||.||$ means the Euclidean distance. The *minimum and maximum scales* $s_{\min}$ and $s_{\max}$, respectively, are chosen so that the points $\mathbf{x}(t)$ and $\mathbf{x}(t')$ are considered to be in a common "infinitesimal" neighborhood. After an *evolution time* $T \in \{1, 2, 3, \ldots\}$, the resulting final distance δ_F is calculated: $\delta_F = ||\mathbf{x}(t+T) - \mathbf{x}(t'+T)||$. Then the local exponential growth rate per time unit is:

$$\lambda_1^{\text{local}} \;=\; \frac{1}{T}\log(\delta_F/\delta_I). \tag{6}$$

To estimate the overall growth rate, in the case of deterministic dynamical systems the largest Lyapunov exponent (LLE) λ_1, the local growth rates are averaged along the trajectory:

$$\lambda_1 \;=\; \langle\lambda_1^{\text{local}}\rangle \;=\; \frac{1}{T}[\langle\log(\delta_F)\rangle - \langle\log(\delta_I)\rangle], \tag{7}$$

where $\langle.\rangle$ denotes averaging over all initial point pairs fulfilling the condition $s_{\min} \leq \delta_I \leq s_{\max}$.

These ideas are applied in the fixed evolution time program for estimating LLE as proposed by Wolf *et al.* [1985]. More details, as well as the code of the program FET1, used in this study, can be found in [Wolf *et al.,* 1985].

The set P of numerical parameters:

$$P \;=\; \{m, \tau, T, s_{\min}, s_{\max}\} \tag{8}$$

is chosen by a user.

The set of 97 baker series with different $\lambda_1(\alpha)$, generated as described in the previous section, is an ideal material for simulating the task of relative characterization, i.e., the task of distinguishing and ordering the series according to their "chaoticity", i.e., according to their λ_1 . The exact dependence of $\lambda_1(\alpha)$ on the parameter α, based on the analytic formula (5), is displayed in Fig. 1a. Figure 1d presents estimates of λ_1 from noise-free data using the following numerical parameters: $m = 2$, $\tau = 2$, $T = 1$, $s_{\min} = 0.01$SD, i.e., 1% of the standard deviation of a particular series, $s_{\max}$ is always defined as $s_{\max} = 10 s_{\min}$ in this study. The λ_1 estimates in Fig. 1d agree with the correct $\lambda_1(\alpha)$ values only for small α, while the majority of the results in Fig. 1d are overestimated. It is possible to "tune" the results by changing some parameters from P (8), e.g., the estimates would decrease using larger evolution time T. Trying to simulate a real problem of classifying experimental time series, where

[1]Thus we concentrate to the chaotic dynamics in the y direction, which is equivalent to a one-dimensional system known as the tilted tent map [Hilborn, 1994].

[2]Note that the GPER (3) is variance-dependent. Therefore all analyzed time series were rescaled in order to have unit variance so that the GPER should classify the series according to their dynamics, without the influence of the variance.

[3]I.e., discrete estimates of the spectral density obtained as squared magnitudes of the Fourier coefficients. The integral in (3) is then computed as a sum over the 8192 periodogram bins.

[4]This shift is equivalent to an addition of white noise to the original time series and thus it could worsen distinction of system states with similar spectra. On the other hand, presence of a few periodogram bins with magnitude close to zero could bias the GPER estimate downwards and obscure the dependence of GPER on a system parameter.

the correct values of λ_1 are unknown (or, in strict mathematical sense they do not exist), it may be dangerous to tune the parameters P for each estimate individually.[5] As the methodologically correct approach we consider using the same parameters P for the whole set of time series, i.e., in each plot of the type of Fig. 1d the estimated LLE's were obtained using the same numerical parameters. The only varying parameter is the parameter α from (4), used in generating the series. Then, we are not interested in absolute values of estimated LLE's, but in relative quantification of different series. In this case, the results can be considered as successful, if a similar curve as that in Fig. 1a was obtained, irrespectively of a scale on the ordinate. The principal shape of the theoretical curve $\lambda_1(\alpha)$ is reproduced by the λ_1 estimates in Fig. 1d. However, the curve is not smooth due to numerical instability of the estimates. Fluctuations of the estimates occur due to a relatively short time series length (1k = 1024 samples) used in the LLE estimation. For a significant decrease of the fluctuations and obtaining smooth curves resembling the theoretical one (Fig. 1a) the series length must be increased by one or two orders of magnitude. We will, however, continue the study using 1k series and consider the results in Fig. 1d as a "good" classification considering "available" amount of data.

In Figures 1e and 1f the same LLE estimates using the same parameters as in Fig. 1d are presented, but the scales $s_{\min} = 0.1$SD and $s_{\min} = 1.0$SD, respectively, were used. The largest Lyapunov exponents λ_1, estimated from the noise-free low-dimensional chaotic series, are stable with respect to different scales (cf. Figs. 1d and 1e), only in the case of the largest scales (Fig. 1f) the estimates have lower values and the curve $\lambda_1(\alpha)$ is partially distorted, but still able to classify the series in the relative sense.

The situation is different when analyzing data with noise. Here we consider additive Gaussian noise added to the data after they have been generated. The term "5% of noise" means that the standard deviation of the added noise is equal to 5% of the SD of the noise free data.

With 5% of noise in the data the classification of the series is practically impossible for $s_{\min} = 0.01$SD (Fig. 2a); possible, though with a higher error rate for $s_{\min} = 0.1$SD (Fig. 2b), while for $s_{\min} = 1.0$SD (Fig. 2c) the results are almost as good as for the noise-free data (Fig. 1f). Thus, the generally known advice that the scales, used in estimating the chaotic measures, should be above the noise level, seems to be valid. Considering, however, that the chaotic measures are defined in terms of vanishing distances between points, one could doubt what is actually measured using the large, macroscopic scales. In this study, is it really the exponential divergence of nearby trajectories, which is reflected in the results in Figs. 1e,f and 2b,c, where the larger scales, i.e., $s_{\min} = 0.1$SD and $s_{\min} = 1.0$SD, respectively, were used?

Searching for an answer, the technique of surrogate data [Theiler *et al.,* 1992], [Paluš, 1995] was used. The surrogate data to an "observed" series are, in this case, realizations of a Gaussian linear stochastic process with the same spectrum as the "observed" series.

For each time series analyzed above, a set of 15 realizations of the surrogates were constructed and the largest Lyapunov exponents λ_1 were estimated using the same parameters P as for the λ_1 of the relevant "observed" series. The results from the surrogates are presented in plots 2d,e,f. (The results from the surrogates related to the noise-free data are practically the same as the results from the surrogates related to the data with the additive noise.)

Exploring relatively small scales ($s_{\min} = 0.01$SD, Fig. 2d), LLE's λ_1 estimated from the surrogates do not reflect the "chaoticity", i.e., the dependence $\lambda_1(\alpha)$ of the original data. Such a result could be expected as far as the chaotic dynamics and nonlinear properties of the original data were destroyed by phase randomization [Theiler *et al.,* 1992], [Paluš, 1995] in the surrogates. Using larger scales $s_{\min} = 0.1$SD and 1.0SD (Figs. 2e and 2f, respectively), however, a relative classification, similar to the ordering of the baker series according to their λ_1, is again observed, though, in the surrogate data there is no exponential divergence of trajectories, or even no trajectories in the deterministic sense! These time series are realizations of *Gaussian linear stochastic* processes, thus their dynamics are fully characterized by their power spectra or, equivalently, by their autocovariance functions. In the previous section we have seen that the "level of chaos" given by LLE λ_1 of the original baker series was reflected also in their linear properties, in particular, the GPER's, obtained from the power spectra, provided the same classsification as the LLE's λ_1 (or KSE). The surrogate data preserve the linear properties of the original data, namely the spectrum and the autocorrelation function, therefore the surrogate data related to the baker series with different LLE's λ_1 can be considered as colored noises with different autocorrelations/spectra. In this situation one can infer that the algorithm for the

[5]This may lead to a subjective bias and false positive results. Even from white-noise data any positive value of the λ_1 estimate may be obtained by tuning the parameters P [Dämmig & Mitschke, 1993].

largest Lyapunov exponent distinguishes linear stochastic time series with different autocorrelation functions. How is it possible?

The LLE algorithm explores changes of initial distances δ_I of pairs of points into final distances δ_F after an evolution time T. Consider a time series generated by white noise (independent identically distributed – IID process). For any initial distance δ_I, the final distance δ_F is a random number independent of δ_I. The averaged $\langle \delta_F \rangle$ is then equal to the overall average distance of the data points. The averaged initial distance $\langle \delta_I \rangle$ is influenced by the choice of the scales $s_{\min}, s_{\max}$. Then, choosing the scales so that $\langle \delta_I \rangle$ is smaller than $\langle \delta_F \rangle$, a positive estimate of λ_1 is obtained. When considered noise is not white but "coloured", i.e., there is some correlation $C(T)$ between $x(t)$ and $x(t+T)$, the increase of distance after the time T is smaller for series with stronger correlations, i.e., the larger $C(T)$, the smaller is the estimated λ_1, and vice versa.

Dämmig and Mitschke [1993] have derived analytic formulae for the λ_1 estimates when applying the considered LLE algorithm to white noise and a very special kind of coloured noise (white noise filtered by a "brickwall filter", the filter function is equal to one for a defined spectral bandwidth, and to zero otherwise). As one could expect, λ_1 estimated from white noise depends exclusively on the parameters P, in the case of the coloured noises λ_1 depends on P and on the spectral bandwidth. Thus for fixed P the estimated Lyapunov exponent λ_1 classifies the series according to their spectra, or, equivalently, according to their autocorrelation functions.

In the case studied here, where the coloured (autocorrelated) noises – the surrogate data – were generated according to given nontrivial spectra, derivation of an analytic formula is probably impossible, however, the dependence of the λ_1 estimates on autocorrelations has been demonstrated above (Figs. 2e,f).

4 Discussion and Conclusion

Almost two decades ago the chaotic measures became frequently used in analysis of complex time series such as the EEG as an alternative to stochastic, mostly linear techniques. Deterministic chaos has been usually considered as an opposite alternative to random effects in attempts to explain complicated dynamics. Recent results indicate, however, that low-dimensional chaos may be rather a rare than ubiquitous phenomenon, especially when considering open, real-world systems, such as those studied in physiology and medicine; or, the strict separation between deterministic-chaotic and stochastic dynamics may be impossible [Ellner & Turchin, 1995]. And even in data generated by a low-dimensional chaotic system, microscopic properties, which are characterized by the chaotic measures, may be unaccessible due to finite precision and measurement noise. In such cases, when the chaotic measures are estimated using scales larger than the noise scale, the chaotic measures do not "measure chaos" anymore, but reflect macroscopic statistical properties of the studied data. In other words, formally successful application of the chaotic measures in a "relative quantification", i.e. an ability to distinguish with a statistical significance EEG signals recorded in different physiological/pathological conditions cannot be considered as an evidence for a chaotic nature of the EEG. The chaotic measures then do not reflect different dimensionality or different rate of exponential divergence of trajectories of hypothetical underlying dynamical systems, but the statistical significances can be caused by a trivial difference in statistical properties of the data sets, such as different autocorrelations, as demonstrated above for the largest Lyapunov exponent estimator, but also by different ranges of data values, different variances, or different noise levels. Then also the statement that the information extracted from EEG signals by using the chaotic measures is entirely new, i.e., independent from information obtained by linear analysis tools, is not correct. Just the opposite has been demonstrated above, that states of chaotic systems can be characterized by information obtained from spectral densities. Therefore the classical spectral analysis should not be underestimated considering nonlinear character of the EEG. The question is just the final "compression" of information contained in estimated spectral densities (periodograms), i.e., whether the conventional spectral bands powers are adequate, or in some cases different quantities should be considered, such as the above entropy rate GPER.

In general, entropy rates (see, e.g., [Cover & Thomas, 1991], [Paluš, 1996b] and references therein), i.e., the rates of information creation by a system, deserve more attention in analysis of complex time series such as the EEG. The entropy rates can be defined for both chaotic and stochastic systems, thus their applications are not jeopardized by possible evidence for or against a particular dynamical mechanism underlying the EEG. Although the exact entropy rate of a continuous system may be unaccessible from data, there is always a possibility to estimate its "coarse-grained" versions, suitable for classification of complex time series [Paluš, 1996b]. Also, the periodogram-based entropy rate

GPER itself could be used as a computationally cheap tool for classification of signals recorded from stochastic or chaotic systems in different states; while discrepancies in the relation between the GPER and a nonlinear entropy rate (Kolmogorov-Sinai entropy or positive Lyapunov exponent or other nonlinear entropy-rate equivalent such as those introduced in [Paluš, 1996b]; a comprehensive review of "complexity" measures, related to entropies and entropy rates can be found in [Wackerbauer *et al.,* 1994]) could be considered for detecting bifurcation onsets in structurally evolving systems [Paluš, 1997a], or, in the EEG context, e.g., for detecting an onset of an epileptic seizure.

Another important task for applications of nonlinear dynamics in EEG analysis is the detection and characterization of nonlinearity and search for its physiological significance by comparing analyses of real EEG data and of signals generated by realistic models. While several studies have been already published (see the references in Introduction) which provide firm evidence for existence of nonlinearity in the EEG, only a few authors yet tried to characterize nonlinear structures found in the EEG (e.g., [Casdagli *et al.,* 1997]), or even to provide comparisons with models (e.g., [Stam *et al.,* this volume]). It might be very interesting to compare results of nonlinear analyses of real EEG data and artificial data generated by complex structural models such as that introduced by Wright & Liley [1996].

The last but not the least remark in this paper is devoted to the quickly developing field of synchronization of chaotic systems. The strongest definition of synchronization requires that the difference between states of synchronized systems asymptotically vanishes. This definition is called *identical synchronization* [Parlitz *et al.,* 1996] while the notion of *generalized synchronization* requires that states of coupled systems are (asymptotically) related by some (possibly complex) function [Rulkov *et al.,* 1995; Kocarev & Parlitz, 1996]. In the classical case of *periodic* self-sustained oscillators, *phase* synchronization is usually defined as locking of phases $\phi_{1,2}$:

$$n\phi_1 - m\phi_2 = \text{const.}, \tag{9}$$

for integer n and m, while the amplitudes can be different. Recently, Rosenblum *et al.* [1996] have discovered the phase synchronization in a case of coupled *chaotic* systems, where the phase entrainment (locking) is described as

$$|n\phi_1 - m\phi_2| < \text{const.}, \tag{10}$$

while the amplitudes of the two systems may be completely uncorrelated, i.e., linearly independent. Considering the field of analysing physiological signals, it is important that many of the results found for the phase synchronization of chaotic systems are valid for stochastic oscillators as well [Rosenblum *et al.,* in press]. The ideas and methods for detection and characterization of the phase synchronization have already found successful applications in analyses of data from cardio-respiratory interaction [Schäfer *et al.* 1998], [Hoyer *et al.* 1998], [Paluš & Hoyer, 1998], and we believe that related methods for the phase synchronization detection [Paluš, 1997b], [Paluš & Hoyer, 1998] can be used in analysing relations of EEG signals recorded from different loci.

In conclusion, we state that although the chaotic measures do not seem to lead to the previously expected progress in the computerized EEG analysis, there are still ideas and tools developed in study of nonlinear (chaotic) systems, which could contribute to understanding the EEG dynamics and underlying brain processes as well as to improvement of clinical diagnostics. We see perspectives for nonlinear dynamics in the computerized EEG analysis namely in detection and characterization of nonlinearity in EEG dynamics, in classifying complexity of the EEG signals by using entropy rates, or in detecting and characterizing synchronization of EEG signals recorded from different loci.

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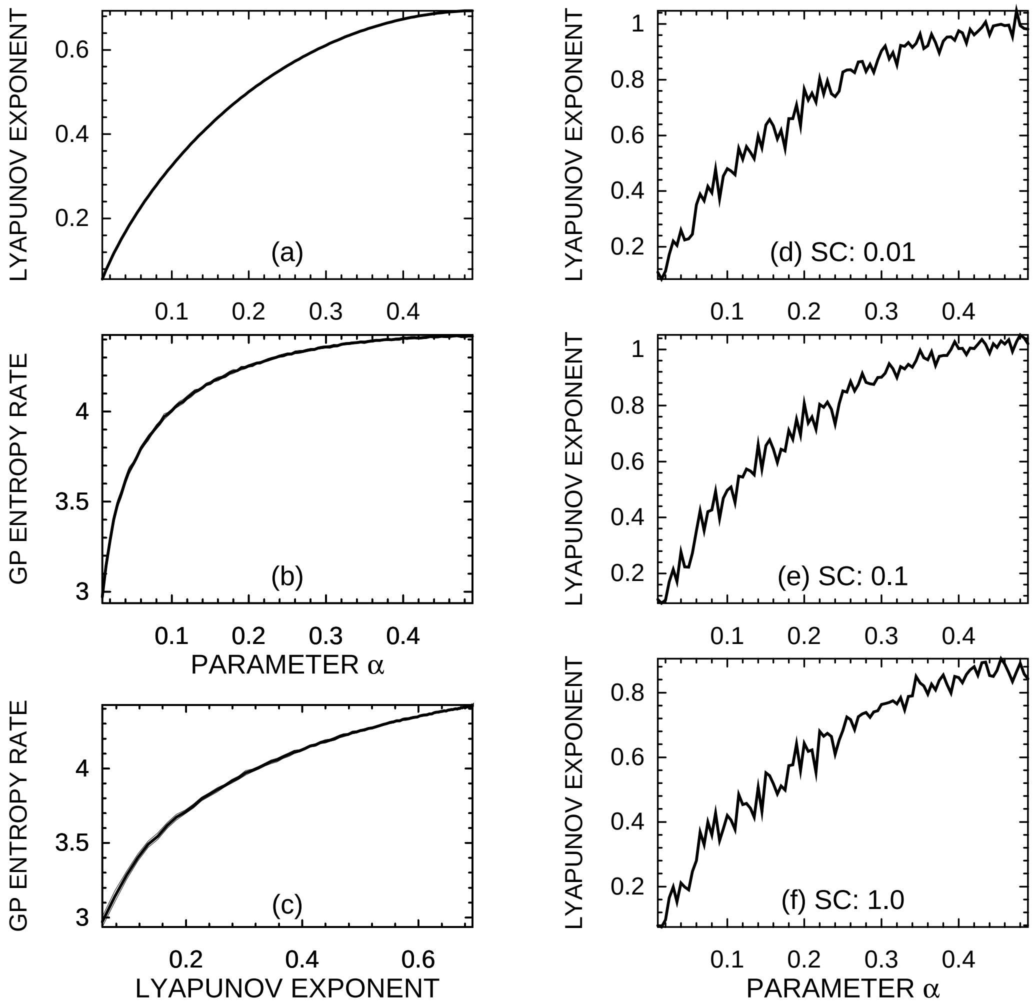

Figure 1: *(a–c) Results from the GPER-LE/KSE relation study for the baker map: a) The Lyapunov exponent as the analytic function of the parameter α. b) The GP entropy rates estimated from 15 realizations of 16k time series (mean – thick line, mean±SD – thin lines, coinciding with the mean) for different values of the parameter α varying from 0.01 to 0.49 by step 0.005. c) Plot of GPER (the same line codes as in b) vs. LE. (d–f) Estimates of the positive Lyapunov exponent from noise-free baker series plotted as functions of the parameter α. The parameters used in estimations were $N = 1024$, $m = 2$, $\tau = 2$, $T = 1$ in all plots, while the scales were defined as follows: $s_{\min} = 0.01SD$ (d), $s_{\min} = 0.1SD$ (e), and $s_{\min} = 1.0SD$ (f).*

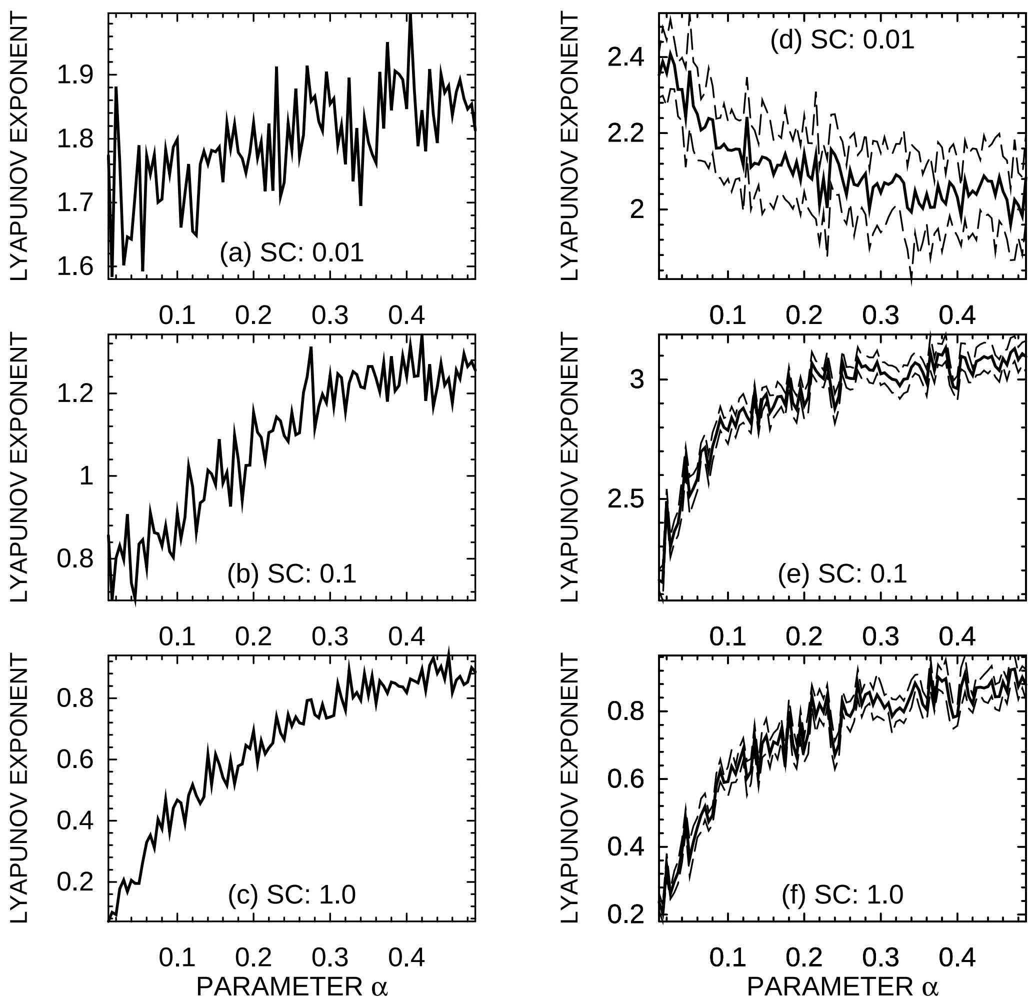

Figure 2: *Estimates of the positive Lyapunov exponent from noisy (5% of noise) baker series (a, b, c) and their surrogate data (d, e, f), plotted as functions of the parameter α. In plots d-e-f solid lines and dashed lines depict mean λ_1 and mean$\pm SD$, respectively, of 15 realizations of the surrogates for each value of α. The scales $s_{\min} = 0.01SD$ (a, d), $s_{\min} = 0.1SD$ (b, e), and $s_{\min} = 1.0SD$ (c, f) were used. The parameters $N = 1024$, $m = 2$, $\tau = 2$, $T = 1$ were used in all estimations.*